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HEAT CONDUCTION AND HEAT EXCHANGE IN TECHNOLOGICAL PROCESSES

TEMPERATURE FIELD OF A SOLID BODY CONTAINING A SPHERICAL HEATING SOURCE WITH A UNIFORMLY MOVING BOUNDARY

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The process of formation of a temperature field in an isotropic solid body containing a spherical heating source with a uniformly moving boundary for the case of realization of nonstationary heat-exchange regimes that lead to the time dependence of the heat-transfer coefficient have been investigated by mathematical-modeling methods.

Keywords: *solid body, spherical heating source, mobility of the boundary, temperature field, integral transformation.*

In the applications of mathematical heat-conduction theory [1–5] investigations of the processes of formation of temperature fields in solid bodies in nonstationary regimes of heat exchange with the external medium that are accompanied by the time variation in the heat-transfer coefficient [4–9] occupy a highly important place. The difficulties arising in solving such problems by analytical methods are well known [4, 10]. They are further aggravated if it becomes necessary to allow for the influence of mechanical and physicochemical processes of different kinds on the temperature field of a thermally stressed solid body. The occurrence of these processes inevitably causes the dimensions of the solid body to change as a consequence of the time variation in the position of its boundaries.

Problems associated with investigation of the temperatures fields in bodies with boundaries moving by a prescribed law [4, 10–15] occupy a special place among problems in the mathematical theory of heat conduction in solid bodies with moving boundaries. Despite the large number of publications, it is unlikely that investigations on the problem in question may be considered completed. In particular, the problem on formation of a temperature field in an isotropic solid body containing a spherical heating source (spherical cavity filled with high-temperature gas, subsequently — with the external medium) with a boundary moving by a prescribed law under time-varying heat-exchange conditions in the studied system remains topical. The investigations carried out in this work seek to solve this problem.

Formulation of the Problem and Mathematical Model. Let us consider an infinite isotropic solid body containing a spherical heating source with a boundary moving by a known law $\nu = \nu(\text{Fo})$ in the regimes of heat exchange with the external medium; these regimes are accompanied by the time variation in the heat-transfer coefficient (nonstationary heat-exchange regimes). Under the assumptions made and in the presence of a central symmetry, a mathematical model of the studied process may be represented in the following form:

$$\frac{\partial \Theta}{\partial \text{Fo}} = \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left(\rho^2 \frac{\partial \Theta}{\partial \rho} \right), \quad \rho > \nu\,(\text{Fo}) > 1\,, \quad \text{Fo} > 0\,;$$

$$\Theta\,(\rho, \text{Fo}) \big|_{\text{Fo}=0} = 0\,; \qquad (1)$$

$$\left. \frac{\partial \Theta\,(\rho, \text{Fo})}{\partial \rho} \right|_{\rho=\nu(\text{Fo})} = \text{Bi}\,(\text{Fo}) \left[\Theta\,(\rho, \text{Fo}) \big|_{\rho=\nu(\text{Fo})} - \zeta\,(\text{Fo}) \right];$$

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$$\rho\Theta(\rho, \mathrm{Fo})\big|_{\mathrm{Fo}>0} \in L^2[\nu(\mathrm{Fo}), +\infty),$$

where $\nu(\mathrm{Fo})$ is the nonnegative nondecreasing function differentiable at least in a generalized sense and $\nu(0) = 1$; according to the solved problem, the functions $\mathrm{Bi}(\mathrm{Fo})$ and $\zeta(\mathrm{Fo})$ may take on only nonnegative values and satisfy the Hölder boundary conditions [16]; the last condition in the mathematical model (1) means that the function $\rho\Theta(\rho, \mathrm{Fo})$ is square-integrable in the radial variable $\rho \in [\nu(\mathrm{Fo}), +\infty)$ for each fixed $\mathrm{Fo} > 0$. Thus, a unique solution of this problem exists in the standard class of functions. We emphasize that finding this solution involves difficulties of a fundamental nature mainly due to the functional dependence $\mathrm{Bi} = \mathrm{Bi}(\mathrm{Fo})$.

In the mathematical model (1), we have

$$\rho = \frac{r}{r_0}, \quad \mathrm{Fo} = \frac{at}{r_0^2}, \quad \Theta = \frac{T - T_0}{T_{\mathrm{m}0} - T_0}, \quad \zeta = \frac{T_{\mathrm{m}} - T_0}{T_{\mathrm{m}0} - T_0}, \quad \mathrm{Bi} = \frac{\alpha}{\lambda} r_0 .$$

We note that in the case of a stationary boundary of a spherical heating source the problem on formation of a temperature field in the studied system has been considered in [1, 9]: in [1], the regime of heat exchange by the Newton law ($\mathrm{Bi}(\mathrm{Fo}) \equiv \mathrm{Bi}$–const) has been analyzed and the solution of the problem has been obtained using the integral Laplace transformation in the variable Fo. Nonstationary (pulsed) heat-exchange regimes have been studied in [9]; here, the approach based on a mixed integral Fourier transform taken in the radial variable $\rho \in [1, +\infty]$ followed by the splitting of its kernel has been used. The thermoelastic reaction of an isotropic solid body with a spherical heating source in thermal-action regimes leading to a time variation in its surface temperature or to convective heat exchange in the system solid body–external medium has been studied in [17].

To simplify further considerations we introduce the function [1]

$$V(\rho, \mathrm{Fo}) = \rho\Theta(\rho, \mathrm{Fo}) \tag{2}$$

and the moving coordinate system

$$X = \rho - \nu(\mathrm{Fo}), \quad \tau = \mathrm{Fo} . \tag{3}$$

Then, according to (1)–(3), the function $V(\rho, \mathrm{Fo})$ is the solution of the following problem:

$$\frac{\partial V}{\partial \tau} = \frac{\partial^2 V}{\partial X^2} + \dot{\nu}(\tau)\frac{\partial V}{\partial X}, \quad X > 0, \quad \tau > 0;$$

$$V(X, \tau)\big|_{\tau=0} = 0;$$

$$\left.\frac{\partial V(X, \tau)}{\partial X}\right|_{X=0} = \varphi(\tau)\, V(X, \tau)\big|_{X=0} - \sigma(\tau); \tag{4}$$

$$V(X, \tau)\big|_{\tau>0} \in L^2[0, +\infty),$$

where

$$\varphi(\tau) = \mathrm{Bi}(\tau) + \nu^{-1}(\tau); \quad \sigma(\tau) = \nu(\tau)\,\mathrm{Bi}(\tau)\,\zeta(\tau) .$$

Method of Solution. To find the solution of problem (4) in analytically closed form we use the singular integral transformation with a parameter p in the variable X [13, 14], which is a generalization of the well-known mixed integral Fourier transformation [18]:

$$u(p,\tau) = \Phi\left[V(X,\tau)\right] \equiv \int_0^\infty V(X,\tau)\, K(X,p,\tau)\, q(X,\tau)\, dX\,, \tag{5}$$

$$V(X,\tau) = \Phi^{-1}\left[u(p,\tau)\right] \equiv \frac{2}{\pi}\int_0^\infty u(p,\tau)\, K(X,p,\tau)\,\frac{p^2 dp}{p^2 + h^2(\tau)}\,, \tag{6}$$

where $\Phi[\bullet]$ and $\Phi^{-1}[\bullet]$ are the operators of the direct and inverse integral transformation (5) and (6) respectively; the kernel $K(X, p, \tau)$ and the weight function $q(X, \tau)$ are determined by the following equalities:

$$K(X,p,\tau) = \exp\left(-\frac{\dot{\nu}(\tau)}{2}X\right)\left\{\cos(pX) + p^{-1}h(\tau)\sin(pX)\right\};$$
$$q(X,\tau) = \exp(\dot{\nu}(\tau)X)\,;\quad h(\tau) = \varphi(\tau) + \frac{\dot{\nu}(\tau)}{2}\,. \tag{7}$$

We note that the kernel $K(X, p, \tau)$ of the integral transformation (5)–(7) is dependent on the variable τ, which makes it impossible to directly use this transformation to seek the solution of problem (4), since in the general case we have

$$\Phi\left[\frac{\partial V(X,\tau)}{\partial\tau}\right] \neq \frac{\partial}{\partial\tau}\Phi\left[V(X,\tau)\right].$$

To overcome the difficulties we use the technique of [13, 14] of splitting the kernel (7) of the integral transformation (5, 6). Introducing the notation

$$A(p,\tau) = \int_0^\infty V(X,\tau)\exp\left\{\frac{\dot{\nu}(\tau)}{2}X\right\}\exp\left\{ipX\right\}dX\,,$$

$$\omega(p,\tau) = 1 - ip^{-1}h(\tau)\,,$$

we can represent the transform of the singular integral transformation in the form

$$u(p,\tau) = \mathrm{Re}\left\{\omega(p,\tau)\,A(p,\tau)\right\}. \tag{8}$$

Here, the identities

$$\Phi\left[\frac{\partial^2 V(X,\tau)}{\partial X^2} + \dot{\nu}(\tau)\frac{\partial V(X,\tau)}{\partial X}\right] \equiv -\left(p^2 + \frac{\dot{\nu}^2(\tau)}{4}\right)u(p,\tau) + \sigma(\tau)\,,$$

$$\Phi\left[\frac{\partial V(X,\tau)}{\partial\tau}\right] = \frac{1}{2}\left[\omega(p,\tau)\int_0^\infty \frac{\partial V(X,\tau)}{\partial\tau}\exp\left\{\frac{\dot{\nu}^2(\tau)}{2}X\right\}\exp\left\{ipX\right\}dX\right.$$

$$\left. + \overline{\omega}(p,\tau)\int_0^\infty \frac{\partial V(X,\tau)}{\partial\tau}\exp\left\{\frac{\dot{\nu}^2(\tau)}{2}X\right\}\exp\left\{-ipX\right\}dX\right],$$

where $\overline{\omega}(p, \tau) = 1 + ip^{-1}h(\tau)$, hold. For the linear law of motion of the boundary of the spherical heating source

$$\nu(\tau) = 1 + 2\beta\tau\,,\quad \tau \geq 0\,, \tag{9}$$

where β is the positive constant, the last identify takes the form

$$\Phi\left[\frac{\partial V(X,\tau)}{\partial\tau}\right]=\mathrm{Re}\left\{\omega(p,\tau)\,\frac{dA(p,\tau)}{d\tau}\right\},$$

which enables us, taking (9) into account, to write the solution of problem (4) in the transforms (8) of the integral transformation (5)–(7) as

$$\mathrm{Re}\left\{\omega(p,\tau)\left[\frac{dA(p,\tau)}{d\tau}+(p^2+\beta^2)\,A(p,\tau)\right]\right\}=\sigma(\tau)\,,\quad \tau>0\,;\quad A(p,0)=0\,. \tag{10}$$

It may be stated that the assumption of linearity of the law of motion (9) of the boundary of the spherical heating source is a sufficient condition for finding the analytically closed solution of problem (4) and for its parametric analysis.

The special property of the Cauchy problem (1) is that it has the set of solutions among which we must single out the unique solution possessing the minimum norm for each fixed $\tau \geq 0$. We can show [3] that with the linear law (9) of motion of the boundary, the sought solution of problem (4) in the transforms (8) of the integral transformation (5)–(7) is written in the form

$$u(p,\tau)=\int_0^{\tau}\frac{p^2+h(\tau)\,h(\xi)}{p^2+h^2(\xi)}\,\sigma(\xi)\exp\left\{-(p^2+\beta^2)(\tau-\xi)\right\}d\xi,\quad \tau\geq 0\,, \tag{11}$$

where

$$h(\xi)=\mathrm{Bi}(\xi)+(1+2\beta\xi)^{-1}+\beta\,;\quad \sigma(\xi)=(1+2\beta\xi)\,\mathrm{Bi}(\xi)\,\zeta(\xi)\,.$$

The analytically closed form of representation of the solution $V(X, \tau)$ of problem (4) follows from (11) if we use the formula of inversion (6) of the singular integral transformation (5)–(7):

$$V(X,\tau)=\Phi^{-1}\left[u(p,\tau)\right],\quad X\geq 0\,,\quad \tau\geq 0\,, \tag{12}$$

where equality (12) is understood in the sense of the standard norm of the linear space $L^2[0, +\infty]$ for each fixed $\tau \geq 0$. To complete the procedure of finding the solution of problem (1) with the linear law (9) of motion of the boundary of the spherical heating source it is sufficient to use equalities (2) and (3). In the moving coordinate system (3), this solution takes the form

$$\Theta(X,\tau)=\frac{2}{\pi(X+1+2\beta\tau)}\exp(-\beta X)\int_0^{\infty}\int_0^{\tau}\frac{p^2+h(\tau)\,h(\xi)}{p^2+h^2(\xi)}\,\sigma(\xi) \tag{13}$$

$$\times\exp\left\{-(p^2+\beta^2)(\tau-\xi)\right\}\left\{\cos(pX)+\frac{h(\tau)}{p}\sin(pX)\right\}\frac{p^2}{p^2+h^2(\tau)}\,d\xi dp\,,\quad X\geq 0\,,\quad \tau\geq 0\,.$$

The temperature of the moving boundary is determined from (13) for $X = 0$:

$$\Theta(0,\tau)=\frac{2}{\pi(1+2\beta\tau)}\int_0^{\infty}\int_0^{\tau}\frac{p^2+h(\tau)\,h(\xi)}{p^2+h^2(\xi)}\,\sigma(\xi)\exp\left\{-(p^2+\beta^2)(\tau-\xi)\right\}\frac{p^2}{p^2+h^2(\tau)}\,d\xi dp\,,\quad \tau\geq 0\,. \tag{14}$$

We note that in practical applications, investigations of the temperature profile $\Theta(0, \tau)$ are of the greatest interest, since they enable us to evaluate the maximum heating of the solid body in the studied (nonstationary) regimes of heat exchange with the external medium.

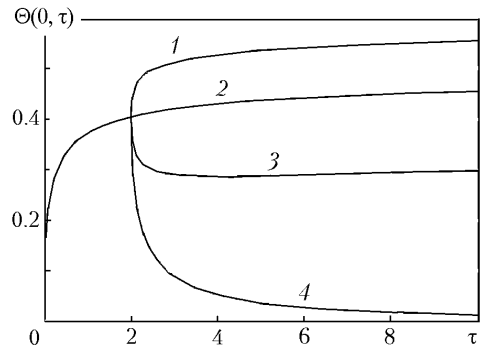

Fig. 1. Temperature profile Θ(0, τ) of the stationary boundary of the solid body with a spherical heating source in the case of realization of different regimes of heat exchange with the external medium: 1) H_2/H_1 = 1.5, 2) 1, 3) 0.5, and 4) 0.

Results and Discussion. We use the obtained theoretical results to investigate the temperature state of an isotropic solid body which contains a spherical heating source with a boundary moving by a prescribed law in pulsed regimes of heat exchange with the external medium; these regimes lead to a step law of variation in Bi = Bi(τ):

$$\mathrm{Bi}\,(\tau) = \begin{cases} H_1\,, & 0 \le \tau \le \tau_*\,; \\ H_2\,, & \tau > \tau_*\,, \end{cases} \tag{15}$$

where $H_1 > 0$ and $H_2 \ge 0$ are constants at constant temperature of the external medium, i.e., setting $\zeta(\tau)$ = 1. This case, along with the available physical interpretation, is important in testing the obtained results, since it leads to the most simple representations of the solution of the initial problem (1)–(3). It is noteworthy that the inequality $H_2 > H_1$ in (15) corresponds to the improvement of the heat-exchange conditions at the boundary X = 0 of the heating source for $\tau > \tau_*$, whereas the inequality $H_2 < H_1$ corresponds to their impairment. The pulsed heat-exchange regime with the duration τ_* of its "active" phase is limiting, which corresponds to the condition H_2 = 0 in (15). When $\tau_* \to +\infty$, or when $H_1 = H_2$, we have the "classical" regime of heat exchange by the Newton law.

Heating Source with a Stationary Boundary. If we have β = 0 and ζ(τ) = 1 in (9), the solution (14) of the initial problem (1) and (2) in the coordinate system (3) for the function Θ(0, τ) in the pulsed regimes of heat exchange (15) with the external medium may be represented in explicit form

$$\Theta\,(0,\tau)\,\big|_{0\le\tau<\tau_*} = \frac{H_1}{H_1+1}\left[1 - \exp\,(h_1^2\tau)\,\mathrm{erfc}\,(h_1\sqrt{\tau})\right],$$

$$\Theta\,(0,\tau)\,\big|_{\tau\ge\tau_*} = \frac{H_1}{h_1+h_2}\left[\frac{H_2}{H_1}\left(1+\frac{h_1}{h_2}\right) - \exp\,(h_1^2\tau)\,\mathrm{erfc}\,(h_1\sqrt{\tau}) - \exp\,(h_2^2\tau)\,\mathrm{erfc}\,(h_2\sqrt{\tau})\right.$$

$$\left. + \exp\left\{h_1^2\,(\tau-\tau_*)\right\}\mathrm{erfc}\,(h_1\sqrt{\tau-\tau_*}) + \frac{1}{h_2}\left\{1 - \frac{H_2}{H_1}\,(h_2+1)\right\}\exp\left\{h_2^2\,(\tau-\tau_*)\right\}\mathrm{erfc}\,(h_2\sqrt{\tau-\tau_*})\right], \tag{16}$$

where $h_k = H_k + 1$, $k \in \{1, 2\}$, and the asymptotic estimate of the studied temperature for high τ values

$$\Theta\,(0,\tau) \sim \frac{H_2}{H_2+1}\left\{1 - \frac{1}{H_2\,(H_1+1)\sqrt{\pi\tau}}\left[H_1 - \frac{H_1-H_2}{(H_2+1)\sqrt{1-\tau^{-1}\tau_*}}\right]\right\} \xrightarrow[\tau\to+\infty]{} \frac{H_2}{H_2+1}$$

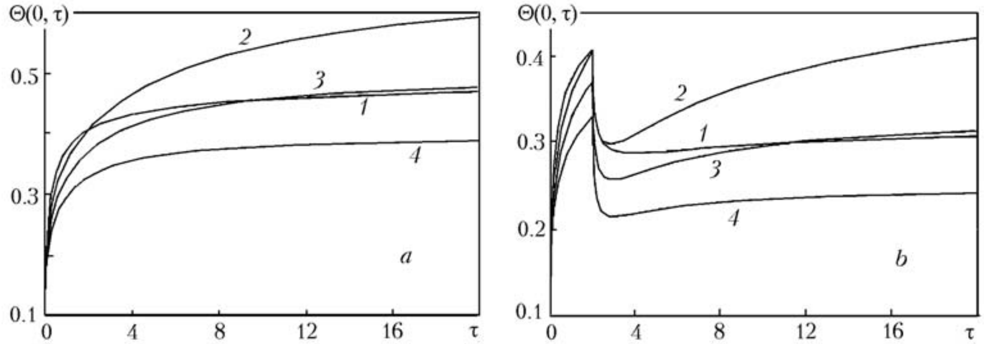

Fig. 2. Comparative analysis of the temperature profiles $\Theta(0, \tau)$ of the stationary and uniformly moving boundaries of the solid body with a spherical heating source in heat exchange by the Newton law (a) ($H_1 = 1$ and $\tau_* \to +\infty$) and in the pulsed regime with impaired conditions of heat exchange with the external medium (b) ($H_2/H_1 = 0.5$ and $\tau > \tau_* = 1$): 1) $\beta = 0$, 2) 0.25, 3) 0.5, and 4) 0.75.

is true. Thus, in pulsed heat-exchange regimes, the value of the maximum temperature is dependent on just the heat-transfer intensity at the boundary $X = 0$ of the heating source. At the same time, the qualitative character of the behavior of the function $\Theta(0, \tau)$ in heat exchange by the Newton law and pulsed heat-exchange regimes is different.

The results of computational experiments partially presented in Fig. 1 ($H_1 = 1$ and $\tau_* = 2$) and the parametric analysis of the solution (16) demonstrate that the improvement of the conditions of heat exchange with the external medium is accompanied by a sharp growth in the temperature at the boundary $X = 0$ of the heating source (curve 1), whereas their impairment involves the formation of the characteristic relaxation zone (curve 3) whose duration is determined by both the duration of the first phase of heat exchange and the modulus of increment of the heat-transfer coefficient; in the pulsed regime of heat exchange with the duration τ_* of its "active" phase, the dependence $\Theta(0, \tau)$ is monotonically decreasing when $\tau > \tau_*$ (curve 4); when $\tau \to +\infty$ we have $\Theta(0, \tau) \to 0$. The above features of the asymptotic (when $\tau \to +\infty$) behavior of the function $\Theta(0, \tau)$ in pulsed regimes of heat exchange with the external medium are attributable to the geometry of the studied system [9].

Heating Source with a Uniformly Moving Boundary. Figure 2 gives partial results of numerical experiments with the use of equality (14) determining the temperature profile $\Theta(0, \tau)$ $\Theta(0, \tau, \beta)$ at the boundary of the spherical heating source with the linear law (9) of the boundary's motion in the regime of heat exchange by the Newton law and the pulsed regime (15) of heat exchange with the external medium, whose realization involves the impairment of heat-exchange conditions when $\tau > \tau_*$, i.e., $(\tau > \tau_*) \Rightarrow (H_1 > H_2)$. Even a superficial analysis of the temperature profiles $\Theta(0, \tau, \beta)$ in Fig. 2 enables us to single out two special properties of the studied process.

1. As the velocity of motion of the boundary of the spherical heating source grows (the value of the parameter β increases) to a certain fixed instant τ_β, the temperature $\Theta(0, \tau, \beta)$ monotonically decreases, which is attributable to the movement of this boundary to a "colder" zone of the solid body.

2. When $\tau > \tau_\beta$, the temperature profile $\Theta(0, \tau, \beta)$ is no longer a monotonic function of the parameter β in the law (9) of motion of the boundary of the spherical heating source. In particular (see Fig. 2), we have $\exists\, \beta_* \in (0, 1)$; $\beta \in [0, \beta_*]$ $\Rightarrow \Theta(0, +\infty, 0) < \Theta(0, +\infty, \beta)$; $\beta = \beta_* \Rightarrow \Theta(0, +\infty, 0) = \Theta(0, +\infty, \beta)$; $\beta \Rightarrow (\beta_*, 1) \Rightarrow \Theta(0, +\infty, 0) > \Theta(0, +\infty, \beta)$. We note that this effect is not observed for the linear heating source with a boundary moving by the linear law. This is most likely due to the geometry of the (spherical) heating source in question. Indeed, for any fixed $\tau > 0$, the shape of the temperature profile $\Theta(0, \tau) \equiv \Theta(0, \tau, \beta)$ is dependent on the specific quantity of heat entering a thermally disturbed zone through the moving boundary of the heating source, which represents a sphere of radius $\rho(\tau) = 1 + 2\beta\tau$ with surface area $S(\tau) = 2\pi\rho^2(\tau)$. In turn this thermally disturbed zone determining the temperature-wave front is a spherical bed of finite length with inner $\rho(\tau)$ and outer $\rho_\beta(\tau)$ radii respectively. This zone's volume is $V(\tau) = \frac{4}{3}(\rho_\beta^3(\tau) - \rho^3(\tau))$. Since the law of motion of the temperature-wave front is nonlinear and the quantity of heat entering the thermally disturbed zone may be regarded, in the

first approximation, as being in proportion to the surface area $S(\tau)$ of the moving boundary of the spherical heating source, the nonmonotonic dependence of the temperature profile $\Theta(0, \tau, \beta)$ on the parameter β with growth in τ becomes clear.

Remark. As has been noted above, the requirement (9) of linearity of the law of motion of the boundary of the spherical heating source is only the sufficient condition of representability of the solution of problem (4) and hence of the initial problem (1) in analytically closed form. To confirm the aforesaid we introduce the moving coordinate system

$$Z = \frac{\rho}{\nu\,(\mathrm{Fo})}\,, \quad \tau = \mathrm{Fo}\,.$$

In this case the initial problem (1) at $\xi(\mathrm{Fo}) = 1$ takes the following form:

$$\nu^2(\tau)\,\frac{\partial\Theta}{\partial\tau} = \frac{\partial^2\Theta}{\partial Z^2} + \left\{\frac{2}{Z} + \frac{Z}{2}\,[\dot{\nu}^2(\tau)]\right\}\frac{\partial\Theta}{\partial Z}\,, \quad Z > 1\,, \quad \tau > 0\,; \quad \Theta(Z, \tau)\big|_{\tau=0} = 0\,;$$
$$\left.\frac{\partial\Theta}{\partial Z}\right|_{Z=1} = \nu(\tau)\,\mathrm{Bi}(\tau)\left[\Theta(Z, \tau)\big|_{Z=1} - 1\right]; \quad Z\Theta(Z, \tau)\big|_{\tau>0} \in L^2\,[1; +\infty)\,. \tag{17}$$

When it is necessary that $[\dot{\nu}^2(\tau)] = C -$ const, we arrive at the following law of motion of the boundary of the spherical heating source:

$$\nu(\tau) = \sqrt{1 + C\tau}\,, \quad C \ge 0\,. \tag{18}$$

To transform problem (17) and (18) to its standard form it is sufficient to take

$$\mathrm{Bi}(\tau) = \nu^{-1}(\tau) \tag{19}$$

and to introduce the new time variable

$$S = \int_0^\tau \nu^2(y)\,dy = 0.5C\tau^2 + \tau\,. \tag{20}$$

Then, according to (18)–(20), problem (17) is equivalent to the following:

$$\frac{\partial\Theta}{\partial S} = \frac{\partial^2\Theta}{\partial Z^2} + \left\{\frac{2}{Z} + \frac{Z}{2}\,C\right\}\frac{\partial\Theta}{\partial Z}\,, \quad Z > 1\,, \quad S > 0\,;$$
$$\Theta(Z, S)\big|_{S=0} = 0\,; \quad \left.\frac{\partial\Theta(Z, S)}{\partial Z}\right|_{Z=1} = \Theta(Z, S)\big|_{Z=1} - 1\,; \quad Z\Theta(Z, S)\big|_{S>0} \in L^2\,[1; +\infty)\,. \tag{21}$$

The solution of problem (21) can be found with either the integral Laplace transformation in the variable S [1–3] or the mixed integral Fourier transformation in the space variable Z [9]. The asymptotics of this solution has the form

$$\Theta(Z, +\infty) = \left\{2\exp\left(-\frac{C}{4}\right) - \frac{\sqrt{\pi C}}{2\sqrt{2}}\,\mathrm{erfc}\,1\right\}^{-1}\left\{\frac{1}{Z}\exp\left(-\frac{CZ^2}{4}\right) - \frac{\sqrt{\pi C}}{2\sqrt{2}}\,\mathrm{erfc}\,Z\right\}$$

and is a function of the parameter C in the law of motion of the boundary of the spherical heating source (18). We note that at $C = 0$, the stationary temperature profile of the solid body is determined as $\Theta(Z, +\infty) = (2Z)^{-1}$, and the temperature of the heating-source boundary is $\Theta(1, +\infty) = 0.5$; when $C \to \infty$, we have $\Theta(1, +\infty) \to 1$.

NOTATION

a, thermal diffusivity, m^2/sec; Bi, Biot number; Fo, Fourier number; erfc (•), complementary Gauss error function; i, imaginary unit; $L^2[0; +\infty]$, linear space of functions square-integrable on a semiinfinite interval $[0; +\infty]$; r, radial variable, m; Re $\{f(z)\}$, real part of the function $f(z)$ of the complex variable z; T, temperature, K; t, time, sec; X, dimensionless space variable of the moving coordinate system; α, heat-transfer coefficient, W/(m^2·K); β, dimensionless positive constant in the linear law of motion of the boundary of the spherical heating source; ζ, dimensionless temperature of the external medium; $\eta(\bullet)$, Heaviside unit function; Θ, dimensionless temperature of the solid body; ν and $\dot{\nu}$, law and velocity of motion of the boundary of the spherical heating source in dimensionless variables (ρ, Fo); λ, thermal conductivity, W/(m·K); ρ, dimensionless radius; τ, dimensionless time of the moving coordinate system; $\Phi[\bullet]$ and $\Phi^{-1}[\bullet]$, operators of the direct and inverse singular integral transformation in the space variable X. Subscripts: m, external medium; 0, initial value.

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